Chapter 5 – Experimental Characterisation of Stiffness of Dry and Lubricated Bearings

# Preface

To qualitatively confirm the increase of bearing stiffness with entrainment velocity,

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# Introduction

Overviews of system identification – Natke, Isermann, Pietrezko, Feng (references in Dietl thesis). Most literature investigate in-situ measurement on operational equipment used in mechanical and aeronautical engineering.

There exist two main methods of system identification: parametric and non-parametric. Non-parametric identification using techniques such as correlation-analysis and Fourier analysis require no physical or modal system parameters to be identified. They provide a good first approximation of system behaviour, useful in applications such as condition monitoring and failure prediction in operating machinery (Kolerus).

Parametric identification requires a representative mathematical model to be established that represents the system under investigation. The system is excited using a defined excitation, and the system response is measured and compared to the response calculated using the mathematical model. Deviation between model and physical system is minimised by adjusting the model parameters. Stiffness and damping coefficients are regarded as physical parameters of the structural model.

The mathematical model can take the form of a finite-element model, continuous models, or rigid multi-body systems. Mult-body systems are advantageous since the computational power required to establish parameter comparisons is much lower than FE or continuous methods. This is due to fewer degrees of freedom representing the physical system. The differential equations of motion describing a linearised system of degrees of freedom is:

Where , , and are the mass, damping and stiffness matrices respectively. The displacement vector is , and is the excitation vector.

Since the aim of these investigations is to qualitatively establish bearing stiffness behaviour with speed and lubricant entrainment, this method was considered sufficient.

Cross-coupled stiffness has greater effect in journal bearings than in rolling element bearings.

# Existing Methodology Literature

**Time-domain parameter identification**

A single degree of freedom system can be simply represented using a free oscillation test. More advanced methods are required when dealing with systems of multiple degrees of freedom, such as in the case of these investigations. Equations of motion for the system are derived as a starting point for the analysis. This method requires all motion in response to excitation (displacement, velocity, and acceleration) to be measured. Parameter estimation methods are then used to establish the unknown quantities of stiffness and damping.

**Frequency-domain parameter identification**

This method identifies unknown parameters of a mathematical model that represents the system using experimentally obtained frequency response functions (FRFs). Arbitrary parameter values are selected for use within the mathematical model, resulting in a difference between the mathematical and experimentally obtained FRFs. The model parameters are then adjusted until the weighted deviation between mathematical and experimental FRFs reaches a minimum value.

Various minimisation methods exist. Linear regression is frequently used. These increase in complexity to deal with system noise, and include the generalised least squares method, the maximum likelihood method, and the instrumental-variable (IVF) method.

**Selecting the appropriate method**

The test rig represents an elasto-mechanical multi-body system. Consensus from literature suggests a preference toward the frequency-domain identification techniques, especially considering the linear nature of the test setup and representative mathematical model. No mass parameters require identification since the values of these are well established from the development of the rig. This allowed for a more stable identification method.

Bearing stiffness and damping independent of frequency? Therefore, only need to measure for one excitation signal. However hammer gives a broadband response.

## Fritzen [1]

Complex stiffness and damping properties of models – numerical models struggle to predict physical. Very sensitive to small parameter changes – small changes lead to big error. Identification algorithms can be used. Inputs to these are either modal data, transfer function or frequency response function, or time-domain data.

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## [2] Y. P. Wang and D. Kim, “Experimental identification of force coefficients of large hybrid air foil bearings,” *J. Eng. Gas Turbines Power*, vol. 136, no. 3, 2014, doi: 10.1115/1.4025891.

Hybrid air foil bearings. Stiffness coefficients measured using time-domain quasi static load-deflection curves and frequency-domain impulse responses. Damping coefficients measured only using frequency responses.

Responses for stiffness for both methods are close to each other. Frequency domain method showed large scatter in identified coefficients with speed, load and supply pressure.

HAFB stiffness and damping characteristics are dependant on excitation frequency,

Frequency domain identification – Intrumental Variable Filter (IVF) method. Essentially a least square method using multivariable regression.

General method as follows:

Equations of motion for a 2DOF rotor dynamics system with unknown stiffness and damping coefficients and mass.

and represent excitation forces in and directions.

Laplace transform given by

Where and represent the Fourier transforms of the , signals and /.

The first terms in the above equation define the system impedance, which consists of the mass, stiffness and damping coefficients of the system. This is then simplified to:

The matrix therefore needs to be identified. All coefficients for stiffness, mass and damping can be factored out as

From the general form of the simplification, measured impedance matrix at each excitation frequency (index ) can be defined as

where is the number of frequency components. The system flexibility matrix, for specific excitation frequency can be found as

Once all system flexibility matrices have been measured for the frequencies of interest, multivariable regression is performed using the augmented matrix accumulated for all the frequencies of interest in the form of the following equation.

where is the accumulation of identity matrix, and is the error to be minimized.

The paper subtracted a baseline dynamic motion due to shaft vibration. It is necessary to remove this in order to extract the true impulse response of the bearing. This is achieved using time-domain subtraction of the baseline signal from the raw impulse response containing the baseline signal after filtering out high-frequency white noise.

## Ophey

## Parameter Identification Method (Notes 14 Param Identification 09)]

***Time-domain measurement methods***

Based on Goodwin [1991] and Nordmann [1980], respectively.

Tiwari, R., Lees, A.W., Friswell, M.I. 2004. “Identification of Dynamic Bearing Parameters: A Review.” The Shock and Vibration Digest, 36, pp. 99-124.

In most cases of parameter identification, methods are restricted to the laboratory environment and limited to rigid rotor configurations and identical bearing supports.

Identification algorithms consider a two degree of freedom representation of the mechanical system under lateral motion. Transmitted forces and resultant motion is measured and test impedances or mobilities are obtained. Often this is done through curve fitting to appropriate transfer functions to provide the parameters of the system.

Lateral motion system of equations:

or

where and is the static equilibrium force required to support the rotor weight.

Force coefficients consist of four stiffness and four damping coefficients for mineral oil lubricated bearings. In liquid annular seals and bearings (hydrostatic and/pr hydrodynamic) working with process fluid (water or LOx), four inertia force coefficients are also important.

The force coefficients () are mechanical parameters which represent a linear or linearized physical system. Considering this, **they must be determined in a test system experiencing only low amplitude motion about equilibrium**. This is often not considered, which is why parameters are vastly different when compared to analytical models. **This method also assumed that coefficients are frequency independent**. This therefore requires a technique to extract results from frequency domain measurements. These parameters are not actually measured, but estimations derived from procedures that relate motion due to applied test forces.

***Frequency-domain measurement methods***

Modern parameter identification techniques use frequency domain procedures. Dynamic force coefficients are estimated from transfer functions of measured displacements due to external loads of a prescribed time-varying structure. Can be used for in-situ measurement as well.

Diagram

Description automatically generated

Consider the bearing as a point mass undergoing forced vibrations induced by external excitation.

Equations of motion for small amplitude excitation about an equilibrium position for a linear mechanical system are

where are external excitation forces, is the test element mass, are any structural support stiffness and resonant damping coefficients (damping from dry system, without lubricant), and, are the bearing dynamic stiffness and damping force coefficients.

Inertia forces are not included. Added mass coefficients not included for bearings. Test system structural stiffness and damping coefficients are obtained from prior shake test results under dry conditions, i.e.. without any fluid through the test bearing.

Two independent force excitations (**impact**, periodic-single frequency, sine-swept, random etc.) and , for example are applied to the test element.

1. Apply and measure ; Apply and measure
2. Obtain the discrete Fourier transform (DFT) of applied forces and displacements

note and

1. For the assumed physical model, the motion ODEs in the frequency domain become

written in matrix form as

Define complex impedances (dynamic complex stiffness) as

where , for ; zero otherwise

These impedances comprise real and imaginary parts, both functions of the excitation frequency, . Real part denotes dynamic stiffness, whilst imaginary party (quadrature stiffness) is proportional to the viscous damping coefficient.

Chart, line chart

Description automatically generated

Real and imaginary parts of ideal mechanism impedance representative of assumed model.

The equations of motion for the first and second tests become,

First test

Second test

Add the two equations and reorganise them.

At each frequency (), the above equation represents four independent equations with four unknows, (),

where

The meaning of linear independence of the test forces (and ensuing motions) is clear. That is, the forces in the second test cannot be a multiple of the first set of forces since then, both the matrix of forces and the matrix of ensuing displacements become singular.

The experimenter must select sets of excitations that are linearly independent, the example  **and**  are preferred (and easy) choices.

In the identification process, the importance of linear independence in the application of forces and ensuing test system or bearings is MOST important to obtain reliable and repeatable results.

In practice, ill conditioned identification matrices can occur even if the measured displacements do not appear similar to each other. The determinant of the matrix is close to zero or is zero. For this case, the condition number of the identification matrix is of importance to determine whether the coefficients that are identified are any good. Isotropic elements excited by a periodic load (single frequency) producing circular orbits of the system usually determine a too ill conditioned system (Murphy 1990).

Curve fitting is used to estimate the system parameters are determined by curve fitting of the test derived discrete set of impedances , one set for each frequency , to the analytical formulas over a pre-selected frequency range. For example,

This method works well for simple curve-fitting of the recorded impedance functions to physical representative analytical functions, i.e., and .

Analytical curve fitting of any data renders a correlation coefficient () representing the goodness of the fit. A low value of the coefficient does not mean the test data or obtained impedance are incorrect, but rather the physical model (analytical function) chosen to represent the test system does not actually reproduce the measurements. To the contrary, a high demonstrates that the physical model, say with constant stiffness and viscous damping C in and respectively, actually describes the measurements (system response) accurately.

Chose analytical function to represent the system. Obtain test data and curve-fit impedance data. Aim for high value.

System transfer functions (output/input) are often used to obtain more precise estimates of seal or bearing force coefficients (Nordmann and Schollhorn, 1980, Massmann and Nordmann, 1985). This process leads to curve fits of non-linear functions.

Transfer functions (displacement/force) known as test system flexibilities are derived as functions of the impedances, from the fundamental i.e.

where

***Instrumental Variable Filter (IVF) method – Fritzen 1985***

Extension of a least-squares estimation method. This is used to simultaneously curve fit all four transfer functions from motion measurements due to two sets of (linearly independent) applied loads. The IVF method has the advantage of eliminating bias typically seen in an estimator due to measurement noise.

The product of the flexibility () and impedance () matrices should be identically equal to the identity matrix since .

However, in any measurement process, there is some noise associated with experiments. Thus, an error matrix () is introduced into the fundamental relationship,

where , , and are matrices of system stiffness, damping and added mass coefficients.

For generality, added mass coefficients () are included in the matrices above.

denotes the measured flexibility matrix, whilst H represents the (to be) estimated test system impedance. This estimated system is the model assumed to best represent the actual test system or element.

In the present method, the flexibility coefficients () work as weight functions of the errors in a minimization procedure. Whenever the flexibility coefficients are large, the error is also penalised by a larger value. As a result, the minimization procedure will become better in the neighbourhood of the system resonances (natural frequencies) where the dynamic flexibilities are maxima (i.e. null dynamic stiffness, (. The measurements containing resonance regions have more weight on the fitted system parameters. External forcing functions exciting the test system resonances are more reliable because at those frequencies the system is more sensitive, and the measurements are accomplished with larger signal to noise ratio.

In addition, it is precisely around the resonant frequencies where all physical parameters (mass, damping and stiffness) most affect appreciably the system response. For “too low” frequencies, the important parameter is the stiffness, while for “too high” frequencies, the inertia dominates the response. Only near resonance do all three parameters have an important effect on the system amplitude response.

It is therefore more accurate to minimize the approximation errors using

rather than a direct curve fitting of impedances, however the procedure leads to a complex minimization scheme.

Write the impedance matrix representing the test system or test element as

with and . Thus, at each discrete frequency ()

Let

therefore

## Dietl Thesis

Identification of equivalent stiffness and damping coefficients - Estimation of frequency response and coherence functions

Based on measured frequency response functions . An FRF describes the ratio between the spectrum of vibration response and the spectrum of the rotor excitation force .

where (\*) indicates complex conjugate quantities. denotes the real-valued auto power spectrum, while is the complex-valued cross power spectrum.

Averaged quantities can be used when estimating the FRFs from several imperfect measurements (pulse takes a rolling average of 3). The response function is estimated as follows (34 from Dietl – Hewlett Packard, The fundamentals of signal analysis, Application Note 243, 1990).

Where represents the average quantities. The power spectra are therefore averaged before computing the FRFs. The coherence function, , is used to indicate the quality of the FRF-estimation:

A value of indicates an undisturbed signal. Signals corrupted by noise will have a value of .

### Logarithmic Decrement

The damping ratio can be obtained from time-domain peak to peak analysis of the decaying impact response. An FFT is first performed to establish frequencies of interest in the system. A cut-off frequency is established, and any noise or signal outside the region of interest is attenuated.

The logarithmic decrement, , is calculated by taking the natural logarithm of the ratio of successive peak amplitudes, , cycles apart.

Diagram

Description automatically generated

Figure - Decaying vibration of linear visco-elastic system

To calculate the logarithmic decrement more accurately, this can be calculated for every successive peak and averaged. Repeating this using different peak separations () allows an average value for the system to be obtained. The following relationships allow the damping ratio, *ζ*, damping, *c*, and natural frequency, *ωn*, to be obtained.

Defined from linearized system

# Experimental Test Rig

## Rig Overview and Modifications

The experimental rig used for initial boundary condition studies [3] was adapted for these investigations. Modifications to the preload mechanism, sensors, and shaft were required to characterize the dynamic and tribological effects of lubricants within the bearings. The rig has been developed such that all components and interfaces are of very high stiffness, with the major source of compliance within the system being the bearings. Deep groove ball bearings, 6205/C2, with an inner and outer diameter of 25 mm and 52 mm respectively sit in test brackets and support the shaft. Critical design considerations are outlined below.

### Shaft

To reduce deflection under radial loading, a 35 mm shaft was manufactured from 16MnCr5 steel. This was selected due to its high tensile and yield strength, high wear resistance and the ability to surface harden in accordance with BS EN ISO 683-3 standard. Bearing seats were manufactured using press-fit collars, allowing different radial preloads to be applied to the bearings depending on which bearings were under tests. For deep groove ball bearings (DGBB), these were designed as a transition fit.

### Axial Load (DGBB)

Axial preload is applied to the bearing as a fixed preload against the outer bearing race. The inner races of both bearings are constrained axially against in shoulder of the shaft, with the outer race being able to displace laterally relative to their fixed position. A bearing cap constrains the motion of the shaft at the motor end of the assembly, exerting a reaction force and hence an equal relative displacement of the races. The preload is applied using either a hydraulic ram, or a screw jack with a load cell in-situ to measure precise preload. Fitment (h7 etc). The piston and load cell design also allows for force data to be acquired simultaneously throughout rig operation.]

Diagram, engineering drawing

Description automatically generated

### Torque Transducer

A RWT421-EC-KG torque transducer was mounted between the motor and shaft. This has a rotational speed rating of 15 000 rpm, and a torque limit of 21Nm with resolution of 0.02% full-scale deflection. This resolution allows for contact level variations in bearing friction to be measured, as well as torque fluctuations from the electric motor. Analog signals for torque and speed are output to the data acquisition chassis, and acquired simultaneously with the radial displacement data.

### Modal Impact Hammer

Impact excitation force was applied to the shaft using a modal hammer t

Impact hammer

Data acquisition rate using B&K Pulse Labshop. Extract the time and frequency domain signals.

### Capacitive Displacement Sensors

Lateral displacement of the shaft in two degrees of freedom were measured using Micro-Epsilon CSH1-CAm1,4 capacitive displacement sensors. These sensors were selected as they provide a non-contact displacement measurement over a 1 mm range to a resolution of ±2 nm; within anticipated shaft deflections calculated from initial experiments and numerical work. Furthermore, due to the conductivity of the shaft material, variations in the material electrical and magnetic properties would not influence measurements, unlike inductive methods such as eddy current sensors.

These sensors were mounted within a collar fitted to the bearing bore in the housing. Displacement measurements obtained are therefore relative between the shaft and housing, showing purely the mechanical runout superimposed on the deviation of the shaft from its nominal geometric centre.

Diagram

Description automatically generated

### Data Acquisition

National instruments

## Equipment

* National Instruments DAQ NIcDAQ.9178 – S/N 168CCE7
* National Instruments DAQ Card – NI9269 – S/N 198920D-01L 1C96632
* National Instruments DAQ Card – NI9215 – S/N 199269A-01L 168EA9F
* DAQ Chassis Power Supply – S/N 2941805
* Charge Amplifier Nexus 4 Channel – S/N 2192450
* Charge Amplifier Power Supply – S/N B21085209000039F
* BNC Voltage Out – AO0087-D-020 2016W35
* BNC Voltage Out – AO0087-D-050 17/08

## Dry vs Lubricated Preliminary Tests

Use acceleration from double differentiation of displacement to find acceleration. Know shaft mass – find components of force due to resonance that we can’t quantify in the system

## Removing Shaft Profile

Electrical runout from magnetic materials such as steel. Localized differences in magnetic fields within the material affect interaction with eddy-current sensors. Capactive displacement sensors do not suffer from this.

Get signal

**Angular position from repeat of peaks take average across number of peaks**

Or do based on angular position – use speed and time data or laser vibrometer

## Sampling Rate

Sampling max 3906.25Hz from the digital output demodulator. The analog voltage signal passes through an antialiasing filter prior to digital conversion.

## Time-Domain Runout Compensation

To study the effects of lubrication on bearings under realistic operating conditions, excitation must be applied to a rotating shaft.

The displacement signal from the shaft has inherent waviness, due in a large part to the machining process. Eccentricity of the bearing seats and variations in rolling element and raceway profile also contribute to this, albeit to a lesser extent. These factors combine to generate mechanical runout of the shaft.

Test this runout using a contacting mechanical dial gauge.

At low speeds with low values of runout, windowing the time signals and averaging the measure FRFs can be used to eliminate the signal disturbance due to runout.

Diagram

Description automatically generated

For high speeds, a more complex method is required to remove the mechanical runout signal. The signals must first be processed using a low-pass filter – to capture the first order oscillations associated withs shaft runout. A signal comparison before and after filtering is demonstrated below. This runout signal was then removed from the time-domain signal by means of vector subtraction. FRFs from this signal absent of runout were then utilized for parameter identification.

The signal of interest is the decaying vibration from the hammer impact. The surrounding and superimposed signal is the runout signal from the rotation of the shaft. Once the shaft returns to steady state following the impact, i.e., the impact response has decayed, the signal returns to that of a pure runout signal.

Precise rotor speed can be calculated from

Low pass Butterworth filter applied. The cut-off frequency was set to to isolate the first order rotational frequency of the signal only.

This technique works up to certain speeds, however around periods of resonance, the

## Total Workflow

Diagram, schematic

Description automatically generated

**Experimental work schematic - Dietl**

**Removal of Runout 20.04.2022**

Import signal

Low pass butterworth filter to 1.2\* rotational frequency

Take a shortened version of the filtered signal and create a time base for this

Find the peaks in this signal which represent maximum runout

Take end portion of signal, isolate and find peaks from this

Create new signals using the nth peak of each one (error minimisation as not all peaks fall perfectly).

Peak distances

Zero crossing point